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1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let
$$p(x) = x^3 + x^2 + x + 1$$

The zero of
$$x+1$$
 is -1. $[x+1 = 0 \text{ means } x = -1]$

$$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$$

= 0

∴By factor theorem, x+1 is a factor of x³+x²+x+1

(ii) $x^4+x^3+x^2+x+1$

Solution:

Let
$$p(x) = x^4 + x^3 + x^2 + x + 1$$

The zero of
$$x+1$$
 is -1. . [$x+1=0$ means $x=-1$]

$$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$

:By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4+3x^3+3x^2+x+1$

Solution:

Let
$$p(x) = x^4 + 3x^3 + 3x^2 + x + 1$$

The zero of x+1 is -1.

$$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$$

:By factor theorem, x+1 is not a factor of $x^4+3x^3+3x^2+x+1$

(iv)
$$x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$$

Solution:

Let
$$p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$$

The zero of x+1 is -1.

$$p(-1) = (-1)^{3} - (-1)^{2} - (2+\sqrt{2})(-1) + \sqrt{2} = -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$$
$$= 2\sqrt{2} \neq 0$$

∴By factor theorem, x+1 is not a factor of $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i)
$$p(x) = 2x^3+x^2-2x-1$$
, $g(x) = x+1$

Solution:

$$p(x) = 2x^3+x^2-2x-1, g(x) = x+1$$

$$g(x) = 0$$

$$\Rightarrow$$
 x+1 = 0

$$\Rightarrow$$
 x = -1

∴Zero of g(x) is -1.

Now,

$$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$

$$= -2+1+2-1$$

$$= 0$$

:By factor theorem, g(x) is a factor of p(x).

(ii)
$$p(x)=x^3+3x^2+3x+1$$
, $g(x)=x+2$

Solution:

$$p(x) = x^3+3x^2+3x+1, g(x) = x+2$$

$$g(x) = 0$$

$$\Rightarrow$$
 x+2 = 0

$$\Rightarrow x = -2$$

$$\therefore$$
 Zero of g(x) is -2.

Now,

$$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$$

$$=-1 \neq 0$$

 \therefore By factor theorem, g(x) is not a factor of p(x).